Bulk scalar field in DGP braneworld cosmology

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Abstract. We investigated the effects of bulk scalar field in the braneworld cosmological scenario. The Friedmann equations and acceleration condition in presence of the bulk scalar field for a zero tension brane and cosmological constant are studied. In DGP model the effective Einstein equation on the brane is obtained with bulk scalar field. The rescaled bulk scalar field on the brane in the DGP model behaves as an effective four dimensional field, thus standard type cosmology is recovered. In present study of the DGP model, the late-time accelerating phase of the universe can be explained .

1. Introduction

In recent times theories with higher-dimensions have received much attention in high energy physics, specially in the context of hierarchy problem, and cosmology [1]. In this scenario it is very likely that our four dimensional universe is a subspace called brane embedded in higher dimensional space-time called bulk. As a realization of such higher-dimensional theory, the brane world scenario has attracted a lot of attention in cosmology. It is believed that the new scenario has the potential to address several issues in cosmology like dark energy, current accelerating phase of the universe, cosmological constant and inflation.

In 2000, Dvali, Gabadadze and Porrati (DGP) introduced a braneworld model of gravity [2]. In this model our universe is a brane embedded in bulk and the standard model particles are confined on the brane and gravity propagates in the bulk (for a review of DGP model see [3]). In DGP braneworld, gravity is modified at large distance rather than at short distance in contrast to other popular braneworld scenarios, because of an induced four-dimensional Ricci scalar in the action. The DGP cosmological model posses a solution which is "self-accelerating" phase. In the studies of DGP model so far, bulk is considered as empty except for a cosmological constant and the matter fields on the brane are considered as responsible for the evolution on the brane.

Braneworld models with non-empty bulk or scalar field in the bulk has been discussed by various authors [4, 5]. It is believed that in the unified theory approach, a dilatonic gravitational scalar field term is required in the the five dimensional Einstein-Hilbert action [6]. One of the first motivations to introduce a bulk scalar field is to stabilize the distance between the two branes [7] in the context of the first model introduced by Randall and Sundrum. A second, motivation for studying scalar fields in the bulk is due the possibility that such a setup could provide some clue to solve the famous cosmological constant problem. Models with inflation driven by bulk scalar field have been studied and it is shown that inflation is possible without inflaton on the brane [5]. Later the quantum fluctuations of brane-inflation and reheating issues are also addressed in the braneworld scenario with bulk scalar field [8]. The creation of a brane world with a bulk scalar field using an instanton solution in a five dimensional Euclidean Einstein equations is also considered [9].

In the present study, we consider the effects of scalar field living in the bulk, on the dynamics of brane. The effective Einstein equations on the brane with a zero tension and its consequence in the braneworld cosmology can be studied. It is possible to show that such a model with bulk scalar field can be used to obtain the standard Friedmann type of cosmology without introducing a tension or cosmological constant. The acceleration conditions for a universe dominated with bulk scalar field is also worth to be examined. The DGP model also has zero tension brane and infinite size extra-dimension, so we are also interested in the DGP model with bulk-scalar field and corresponding cosmological solutions. The Friedmann type cosmology can be obtained in the DGP braneworld cosmology, under suitable conditions, without matter field on the brane. It can be

shown that, in our current frame work of the DGP braneworld, the solution of Einstein equation exhibit a self accelerating phase of the universe. The rescaled bulk scalar field on the brane in the DGP model is expected to mimic the inflaton on the brane and thus standard cosmology could be recovered.

2. Effective Einstein equation for tension free brane

Braneworld models without/with a cosmological constant [10, 11] and infinite size flat extra dimensions have been received with much attention recently [12]. Consider a three dimensional brane embedded in a five dimensional bulk and assume that the five dimensional metric have the following form

$$ds^2 = g_{AB}dx^A dx^B = dy^2 + q_{\mu\nu}dx^\mu dx^\nu. \tag{1}$$

Here onwards the Latin indices are running from 0 to 4 and Greek indices are from 0 to 3.

The Einstein equation for the metric (1) with source can be written as (with $k_5^2 = 8\pi G_5$)

$$R_{AB} - \frac{1}{2}g_{AB}R = -\Lambda g_{AB} + k_5^2(T_{AB} + S_{AB}\delta(y)). \tag{2}$$

Where T_{AB} is the five dimensional energy-momentum tensor and S_{AB} denote the energy momentum tensor confined on the brane. We assume that the brane is located at $y = 0$ in the bulk, the Einstein's equation on brane is given by [13],

$$\begin{aligned}
G_{AB} = \frac{2k_5^2}{3}[(T_{CD} - \frac{\Lambda}{k_5^2}g_{CD})\ q_A^B q_B^A + ((T_{CD} - \frac{\Lambda}{k_5^2}g_{CD})n^C n^D \\
- \frac{1}{4}(T^C\ _C - \frac{\Lambda}{k_5^2}g_{CD}))q_{AB}] - E_{AB} + KK_{AB} - K\ _A^C K_{BC} \\
- \frac{1}{2}q_{AB}(K^2 - K^{CD}K_{CD}),
\end{aligned} \tag{3}$$

where g_{AB} and q_{AB} are related as

$$g_{AB} = q_{AB} + n_A n_B, \tag{4}$$

and n_A is a unit vector normal to the brane.

In the present study, we assume that bulk is empty and also the bulk cosmological constant Λ is considered to be absent. The energy-momentum tensor on the brane with a zero tension takes the following form

$$S_{AB} = \tau_{AB}. \tag{5}$$

Where τ_{AB} represents the matter fields confined on the brane and is assumed to have a perfect fluid form. The term E_{AB} appearing in the equation (3) is related to the five dimensional Weyl curvature tensor C_{ABCD} ($E_{AB} = C_{ABCD}n^C n^D$) and K_{AB} is the extrinsic curvature associated with the brane. Assuming that bulk space-time is Z_2 symmetric with respect to the brane, the Israel Junction condition [14] implies

$$[K_{\mu\nu}] = -k_5^2(S_{\mu\nu} - \frac{1}{3}q_{\mu\nu}S). \tag{6}$$

Using the condition (6) in (3) one obtains the effective Einstein equation on the brane,

$$G_{\mu\nu} = k_5^4 \Pi_{\mu\nu} - E_{\mu\nu}. \tag{7}$$

Where,

$$\Pi_{\mu\nu} = -\frac{1}{4}\tau_{\mu\rho}\tau_\nu^\rho + \frac{1}{12}\tau\tau_{\mu\nu} + \frac{1}{8}q_{\mu\nu}\tau_{\alpha\beta}\tau^{\alpha\beta} - \frac{1}{24}q_{\mu\nu}\tau^2. \tag{8}$$

An alert reader can immediately realise that the Friedmann equation arising from equation (7) will be of the form $H \propto \rho$. Thus recovery of the standard cosmology is impossible, which is not compatible with cosmological observations. This problem is due to absence of cosmological constant in bulk and tension on the brane, a point noted earlier by [15, 16]. We show, in the next section, this problem can be solved in the braneworld cosmology with a non-empty bulk.

2.1. Effective Einstein equation with bulk scalar field

In this section we study effective Einstein equations in presence of the bulk scalar field and see effects in the braneworld cosmology.

We introduce a scalar field in the bulk and cosmological constant is still assumed to be zero, thus the corresponding five dimensional Einstein's equation take the following form

$$R_{AB} - \frac{1}{2}g_{AB}R = k_5^2(T_{AB} + S_{AB}\ \delta(y)). \tag{9}$$

Where T_{AB} is the energy momentum tensor of the bulk scalar field and is given by

$$T_{AB} = \phi_{,A}\ \phi_{,B} - g_{AB}(\frac{1}{2}g^{CD}\phi_{,C}\ \phi_{,D} + V(\phi)). \tag{10}$$

The energy momentum tensor on the brane with zero tension is same as (5) and the induced four dimensional Einstein's equation on brane i.e $y = $ constant (zero) hyper surface is given by equation (3).

Again assuming Z_2 symmetry for bulk space time [14], using the junction condition (6) and equation (10), effective Einstein on the brane becomes,

$$G_{\mu\nu} = k_5^2\hat{T}_{\mu\nu} + K_5^4\Pi_{\mu\nu} - E_{\mu\nu}. \tag{11}$$

Where,

$$\hat{T}_{\mu\nu} = \frac{1}{6}\left(4\phi_{,\mu}\ \phi_{,\nu} + \left(\frac{3}{2}(\phi_{,y})^2 - \frac{5}{2}q^{\alpha\beta}\phi_{,\alpha}\ \phi_{,\beta} - 3V(\phi)\right)q_{\mu\nu}\right), \tag{12}$$

and Π is same as given by equation (8) and $E_{\mu\nu}$ is the part of the Weyl tensor.

Here also one can see that H^2 is not linear in the energy density of brane matter. Since the bulk scalar field contributes a term linear in its energy density, we can recover standard type cosmology due to it.

2.1.1. Friedmann equation The purpose of this section is to get Friedmann like equations on the brane. We are interested in homogeneous and isotropic geometries on the brane, hence the metric on the brane is taken as the Friedmann-Robertson-Walker metric,

$$ds^2|_{y=0} = -dt^2 + S^2(t)\delta_{ij}dx^i dx^j. \tag{13}$$

The matter content on the brane is assumed to be perfect fluid form and satisfies the usual energy-momentum conservation law. The bulk scalar field under our consideration satisfies the following boundary condition

$$\phi, y|_{y=0} = 0, \tag{14}$$

which means that bulk scalar is constant with respect to y on the brane. Using equations (11), (12) and (13), the Friedmann equation on the brane is obtained as,

$$\left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} = \frac{k_5^2}{3}\rho_B + \frac{k_5^4}{36}\rho_b^2 - \frac{E_{00}}{3}. \tag{15}$$

Where ρ_b is the brane energy density and ρ_B is the bulk energy density and can be obtained from (12) as

$$\rho_B = \frac{1}{2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right). \tag{16}$$

Next, consider a model in which the brane field and dark radiation terms (E_{00}) on the right side of Friedmann's equation are negligible, then Eqn 15 becomes

$$H^2 + \frac{k}{S^2} = \frac{k_5^2}{3}\rho_B, \tag{17}$$

which implies that dynamics on the brane are governed by the bulk scalar field.

Next, we are interested in the inflationary phase driven by the bulk scalar field rather than due to inflaton on the brane. Consider a universe where the bulk scalar field dominates, the equation governing the scalar field is the Klein-Gordon equation. Using the full five dimensional metric (1) with a Friedmann- Robertson-Walker metric for brane part, the Klein-Gordon equation is obtained as,

$$\ddot{\phi} + 3\left(\frac{\dot{S}}{S}\right)\dot{\phi} - \partial_y^2\phi + V' = 0. \tag{18}$$

In view of the slow-roll approximation the Klein-Gordon equation on the brane becomes,

$$3H\dot{\phi} + V' = 0. \tag{19}$$

The condition for acceleration can be found from (17), by assuming that the bulk scalar field follow the usual energy momentum conservation law for the bulk matter on the brane i.e, $\dot{\rho}_B = -3H(P_B + \rho_B)$, as:

$$\frac{\ddot{S}}{S} = -\frac{k_5^2}{6}(\rho_B + 3P_B) > 0 \tag{20}$$

from this one obtains,

$$P_B < -\frac{\rho_B}{3} \tag{21}$$

which matches the acceleration conditions of the standard cosmology.

Our next goal is to obtain the acceleration condition for a universe with scalar field in the bulk. For this, the energy density of bulk scalar field can be considered as the form given by equation (16) and pressure is obtained from (12) as $P_B = \frac{5}{2}\dot{\phi}^2 - 3V$. Thus, in the scalar field dominated universe the acceleration condition can be derived using equation (21) as

$$\dot{\phi}^2 < \frac{2}{3}V. \tag{22}$$

Which is similar to the standard acceleration condition except for a factor difference. This indicate that the bulk scalar field need more potential energy than the standard inflaton field for inflation to occur.

3. Bulk scalar field in DGP braneworld

In the DGP model our universe a is a three brane embedded in five dimensional bulk and there is an induced four dimensional Ricci scalar on the brane, due to radiative correction to the graviton propagator on the brane. In this model there is a length scale below which the potential has usual Newtonian form and above which the gravity becomes five dimensional. The cross over scale between the four dimensional and five dimensional gravity is

$$r_c = \frac{k_5^2}{2\mu^2}. \tag{23}$$

Where $\mu^2 = 8\pi G_4$. We start with a generalised DGP model in which both bulk cosmological constant Λ and brane tension σ are non-zero. In our model we consider a non-empty bulk, with a bulk scalar field living in it. The bulk energy momentum tensor is given by equation (10) and the four dimensional energy-momentum tensor given by

$$S_{AB} = \tau_{AB} - \sigma q_{AB} - \mu^{-2} G_{AB}. \tag{24}$$

Where last term in the above equation represents the induced term. Again assuming bulk is Z_2 symmetric, the Einstein equation on the brane is obtained as:

$$\left(1 + \frac{\sigma k_5^2}{6\mu^2}\right) G_{\mu\nu} = -\left(\frac{k_5^2 \Lambda}{2} + \frac{k_5^4 \sigma^2}{12}\right) q_{\mu\nu} + \mu^2 \tilde{T}_{\mu\nu}$$
$$+ \frac{\sigma k_5^4}{6} \tau_{\mu\nu} + \frac{k_5^4}{\mu^4} F_{\mu\nu} + k_5^4 \Pi_{\mu\nu} + \frac{k_5^4}{\mu^2} L_{\mu\nu} - E_{\mu\nu}, \tag{25}$$

where $\tilde{T}_{\mu\nu}, \Pi_{\mu\nu}, F_{\mu\nu}$ and $L_{\mu\nu}$ are respectively given by,

$$\tilde{T}_{\mu\nu} = \frac{r_c}{3}\left[\left(4\phi_{,\mu}\,\phi_{,\nu} + \left(\frac{3}{2}(\phi_{,\chi})^2 - \frac{5}{2}q^{\alpha\beta}\phi_{,\alpha}\,\phi_{,\beta} - 3V(\phi)\right)q_{\mu\nu}\right)\right], \tag{26}$$

$$\Pi_{\mu\nu} = -\frac{1}{4}\tau_{\mu\rho}\tau_\nu^\rho + \frac{1}{12}\tau\tau_{\mu\nu} + \frac{1}{8}q_{\mu\nu}\tau_{\alpha\beta}\tau^{\alpha\beta} - \frac{1}{24}q_{\mu\nu}\tau^2, \tag{27}$$

$$F_{\mu\nu} = -\frac{1}{4}G_{\mu\rho}G_\nu^\rho + \frac{1}{12}GG_{\mu\nu} + \frac{1}{8}q_{\mu\nu}G_{\alpha\beta}G^{\alpha\beta} - \frac{1}{24}q_{\mu\nu}G^2, \tag{28}$$

$$L_{\mu\nu} = \frac{1}{4}(G_{\mu\rho}\tau_\nu^\rho + \tau_{\mu\rho}G_\nu^\rho) - \frac{1}{12}(\tau G_{\mu\nu} + G\tau_{\mu\nu}) - \frac{1}{4}q_{\mu\nu}(G_{\alpha\beta}\tau^{\alpha\beta} - \frac{1}{3}G\tau) \tag{29}$$

As mentioned earlier $E_{\mu\nu}$ is the projection of the Weyl tensor on the brane. The Einstein equation in DGP cosmology is differrent from equations (7, 11) as it contains extra term, which is quadratic in $G_{\mu\nu}$ due to inclusion of $G_{\mu\nu}$ in the four dimensional energy momentum tensor.

At this stage we switch to the original DGP model where cosmological constant and brane tension are zero then equation (25) becomes,

$$G_{\mu\nu} = \mu^2 \tilde{T}_{\mu\nu} + \frac{k_5^4}{\mu^4} F_{\mu\nu} + k_5^4 \Pi_{\mu\nu} + \frac{k_5^4}{\mu^2} L_{\mu\nu} - E_{\mu\nu}. \tag{30}$$

3.1. The Friedmann's equation in DGP cosmology

In this section we discuss the Friedmann type equations, for this the universe is considered homogeneous and isotropic on the brane. The metric on the brane is the Friedmann-Robertson-Walker metric (13), taking the 00 component of the equation (30) becomes

$$G_{00} = \mu^2 \rho_B + \frac{k_5^4}{12} \rho_b^2 + \frac{k_5^4}{12\mu^4} G_{00}^2 - \frac{k_5^4}{6\mu^2} \rho_b G_{00} - \frac{\mathcal{E}}{S^4} \tag{31}$$

which can be rewritten as

$$\left(H^2 + \frac{k}{S^2} \right) = \epsilon \frac{2\mu^2}{k_5^2} \sqrt{ \left(H^2 + \frac{k}{S^2} \right) - \frac{\mu^2}{3} \rho_B - \frac{\mathcal{E}}{S^4} } + \frac{\mu^2}{3} \rho_b. \tag{32}$$

This is the general Friedmann equation in DGP cosmology and it matches with the result obtained in [17]. The $\epsilon = \pm 1$ corresponds to two possible embedding of the brane in the bulk. Consider a case in which μ go to inifty i.e induced curvature term is not there (and $\mathcal{E}$ is negligible) then equation (32) becomes

$$H^2 + \frac{k}{S^2} = \frac{\mu^2}{3} \rho_B + \frac{k_5^4}{36} \rho_b^2, \tag{33}$$

which is the Friedmann equation obtained in (15). Hence when the curvature term is not included in S_{AB} (or in action) our result reduces to the flat case and is same as in [11, 13].

3.2. Standard cosmology with a bulk scalar field

The standard cosmology is recovered from DGP braneworld scenario, with a brane matter by applying suitable conditions [17]. However we are interested in seeing the dynamics of brane due to a bulk scalar field and to obtain standard type cosmology with bulk field. We assume that brane has no matter and also $\mathcal{E}$ is zero, then equation (32) becomes,

$$\left(H^2 + \frac{k}{S^2} \right) \frac{k_5^2}{2\mu^2} - \epsilon \sqrt{ \left(H^2 + \frac{k}{S^2} \right) - \frac{\mu^2}{3} \rho_B } = 0. \tag{34}$$

We get standard type cosmology from the above equation by imposing the condition that first term in (34) should be negligible i.e,

$$\left(H^2 + \frac{k}{S^2}\right)\left(\frac{k_5^2}{2\mu^2}\right)^2 \ll 1 \tag{35}$$

thus,

$$H^2 + \frac{k}{S^2} = \frac{\mu^2}{3}\rho_B. \tag{36}$$

Note that the condition (35) for recovery of standard cosmology is different from one obtained in [17] with matter field on the brane. The bulk energy density is given by,

$$\rho_B = r_c(\frac{1}{2}\dot{\phi}^2 + V(\phi))$$
$$= \frac{1}{2}\dot{\Phi}^2 + \tilde{V}(\Phi). \tag{37}$$

Where we introduced the effective rescaled field Φ on the brane,

$$\Phi = \sqrt{r_c}\phi \tag{38}$$

and the potential is rescaled as,

$$\tilde{V}(\Phi) = r_c V(\frac{\Phi}{\sqrt{r_c}}). \tag{39}$$

The interesting feature of the effective field Φ is that, the rescaling depends on the cross over scale. Our study of the rescaling the scalar field and hence the potential is without taking a specific form. However one can consider different forms of the potential and check that the resclaing holds. The rescaled bulk scalar field on the brane act as an effective field and thus the dynamics of the brane can be goverened by it. For a universe dominated by a bulk scalar field, the condition for acceleration in terms of the rescaled field, on the brane is obtained as,

$$\dot{\Phi}^2 < \frac{2}{3}\tilde{V}. \tag{40}$$

As mentioned earlier the only modification coming is a factor compared to the standard inflation.

In view of the rescaling one can immediately infer that, following the relation (38), the cross overscale can be written in the present context as

$$r_c = \frac{\Phi^2}{\phi^2}. \tag{41}$$

3.3. Late time cosmology and solutions to Friedmann equation

The most important aspect of DGP model is the self accelerating solutions in the late universe. Let us see, the presence of bulk scalar field can give rise self acceleration in DGP model. For this, consider the two branches of solution of the effective Friedmann

equation (32) which depend upon the value of ϵ and in absence of ρ_b it can be rewritten as,

$$H^2 + \frac{k}{S^2} = \frac{1}{2r_c^2} \left[1 + \epsilon \sqrt{1 - \frac{4\mu^2}{3} \rho_B r_c^2} \right]. \tag{42}$$

As mentioned earlier, the bulk scalar field on the brane satisfies the usual energy momentum consevation law and hence it follow that $\rho_B \propto S^{-3}$. Thus equation (42) can be expanded under the condition that $\mu^2 \rho_B \ll 1/r_c^2$. At zeroth order for $k = 0$ case, the two branches are

● $\epsilon = -1$

$$H^2 = 0 \tag{43}$$

correspond to a Friedmann universe with $S \sim$ constant, which means that it is asymptotically static.

● $\epsilon = 1$

$$H^2 = \frac{1}{r_c^2} \tag{44}$$

which is the self-accelerated solution in agreement with [17], and $S \sim \exp(t/r_c)$, which shows the self-accelerating phase of the universe. Therefore, we can consider a model of the universe that filled with scalar field in bulk and can lead to a self accelerating phase of the universe as in the case of the cosmolgical models with inflaton or matter field in the brane. Thus the late time behaviour of the universe does not alter even if one start a cosmological model with scalar field living in the bulk without matter content in the brane. Hene the effective rescaled bulk scalar field may be an alernative to the inflaton on the brane.

4. Conclusions

We examined the effects of bulk scalar field in the braneworld cosmological scenario. Our basic set up is a three dimensional brane embedded in a five dimensional space-time with an infinite size extra-dimension. We derived the effective Einstein equations on the brane for different cases.

It is noted that the effective Einstein equation, in the case of zero tension brane and cosmological constant, leads to Friedmann equation with $H \propto \rho$ rather than usual situation where $H \propto \sqrt{\rho}$. Therefore the aformetioned case cannot lead to the standard type of cosmology, which confronts with cosmological observations so far. In order to overcome this unlikely situation, one has to introduce a bulk cosmological constant and brane tension in a braneworld cosmological model under the consideration. As an alternative, we show that the introduction of scalar field in the bulk can evade this problem. We have shown that the derived Einstein's equation can recover the standard Friedmann equation with a bulk scalar field in absence of the brane tension and cosmological constant. Using this model inflation driven by the bulk scalar field on

the brane is also examined and the derived conditions for acceleration are matching with the standard case except for a factor difference. This is due the modification coming from the extra-dimensional effect. It further indicates that the required potential energy due to the effective bulk scalar field is much higher than that of the inflaton on the brane for inflation to occur.

The Einstein equation in DGP cosmology are also derived, then we recovered the standard type of cosmology in terms of the bulk scalar field. The condition for recovery of standard cosmology is different from the one obtained with a brane field. And it shows that condition is reversed in comparision with [17], interestingly still H goes as r_c^{-1}. We introduced a rescaled bulk scalar field on the brane in the DGP model and the rescaling depends on scale factor r_c. Though we have introduced the rescaling of the potential in a general manner, the validity of the rescaling procedure can be examined by taking specific forms of the potential. The DGP model with the rescaled bulk scalar field on the brane can also provide a self accelerating solution. Thus our results has concordance with Deffayet's result [17] in the late-time behaviour of the universe. Therefore the rescaled bulk scalar field evaluated on the brane can indeed replace the brane field and can govern the dynamics on the brane.

The rescaling procedure of the field helps us to express the crossover scale as the ratio of the effective scalar field to the bulk field. Although we have considered brane as empty in this work, it would be interesting to retain the brane field and consider a possible bulk-brane fields interactions. One could then examine whether this system address the issue of alternative to dark energy and thus test the viability of this model with respect to observations.

References

[1] P. Horava and E. Witten, Nucl. Phys. B **460**, 506 (1996), Nucl. Phys. B **475**, 94 (1996), P.Brax and C. Bruck, Class.Quant.Grav.**20** R201 (2003).

[2] G. Dvali, M. Gabadadze and M. Porrati, Phys.Lett. B **485**, 208 (2000).

[3] A. Lue, Phys.Rep.**423**,1 (2006).

[4] D. Langalois and M. Rodriguez-Martinez Phys. Rev.D **64**, 123507 (2001), S.C. Davis, JHEP 0203, 058 (2002), J. Ren and X. Meng Phys.Lett.**B636** 5 (2006), E. Flanagan, S.H. Tye and I Wasserman, Phys.Lett.B **522**, 155 (2001).

[5] Y. Himemoto and M. Sasaki Phys. Rev.D **63**, 044015 (2001).

[6] K. Maeda and D. Wands, Phys. Rev.D **62**, 124009 (2000).

[7] P. Kaonti, K Olive and M Pospelov, Phys.Lett. B **481**, 386 (1999).

[8] J. Yokoyama and Y. Himemoto, Phys. Rev. D **64**, 083511 (2001).

[9] H. Aoyanagi, K. Maeda, JCAP 0603, 012 (2006).

[10] P. Binetury, C. Deffayyet and D. Langlois, Nucl.Phys.B **565**, 269 (2000).

[11] P. Binetury, C. Deffayyet, U. Ellwanger and D. Langlois, Phys.Lett.B **477**, 285 (2000).

[12] B. Gumjudpai, Gen. Rel. and Grav. **36**, 747 (2004).

[13] T. Shiromizu, K. Maeda and M. Sasaki, Phys. Rev.D **62**, 024012 (2000).

[14] W. Israel. Nuovo Cimento **44B**, 1 (1966).

[15] J. M. Cline, C. Grojean and G. Servant, Phy. Rev. Lett. **83**, 4245 (1999).

[16] P. Kaonti, I. Kogan, K. Olive and M. Pospelov, Phys.Lett. B **468**, 31(1999).

[17] C. Deffayet, Phys.Lett.B **502**, 199 (2001).